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FLUID PROPERTY VARIATION ANALYSIS IN A HEAT EXCHANGER USING SUPERCRITICAL FLUIDS

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ABSTRACT

A Supercritical fluid is a fluid held at or above its critical temperature and critical pressure, where distinct liquid and gaseous phases do not exist. Carbon dioxide behaves as a supercritical fluid above its critical temperature (304.25 K) and critical pressure (72.9 atm). The thermophysical properties of CO_2 change significantly with temperature and pressure in the supercritical region. At a considered pressure value, the thermal conductivity, dynamic viscosity and density increases with the increase in temperature. This paper aims to investigate the performance of a two-fluid co-current and counter co-current flow heat exchanger, with one of the interacting fluids as supercritical carbon dioxide. The governing equations are written for one dimensional flow of the fluids. For simplicity, only the variation of the isobaric heat capacity of supercritical CO_2 with temperature along the heat exchanger length is considered. The equations are solved by finite element method using MATLAB software. The performance of the heat exchanger is analyzed using temperature plots and effectiveness-NTU method. The performance of heat exchanger is found to be enhanced with the use of supercritical fluids when compared with that of a regular fluid (water).

KEY WORDS: **S**uper critical fluids, two-fluid heat exchanger, and finite element method.

1. INTRODUCTION

A heat exchanger is a device used for exchanging energy between two or more fluids at different temperatures. Heat exchangers can be applied in many industries such as power, electronics, refining, cryogenics, chemicals, metals and manufacturing sectors. Supercritical fluids are used extensively in areas of refrigeration and cryogenics owing to their cheap, chemically inert, non-toxic and non-flammable nature. In the present analysis, a two-fluid heat exchanger of co-current and counter co-current arrangements is considered, introducing one of the fluids as a supercritical fluid (sCO_2). Fig.1 and Fig.2 are the schematic representations of the direction of flow of the hot and cold fluid streams in the respective arrangements.

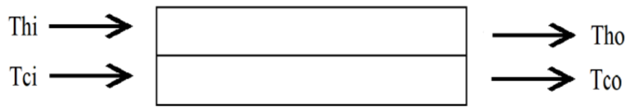

Fig.1 Schematic of co-current flow two-fluid heat exchanger

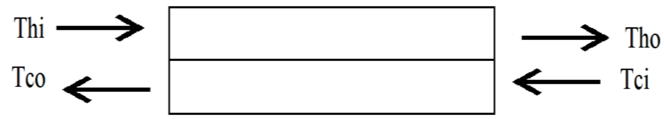

Fig. 2 Schematic of counter co-current flow two-fluid heat exchanger

Lazova M et al. [1] worked on the applications of supercritical fluid in a heat exchanger suitable for organic Rankine cycles. They mentioned the work done on supercritical fluid flows in a heat exchanger and elaborated on the prospective areas of research on supercritical fluids. Pettersen et al. [2-3] measured the heat transfer co-efficient and pressure drop in a flat, multiport, extruded Aluminum tube under cooling conditions. Their work compares the measured heat transfer co-efficient with several single-phase calculation models. Simoes et al. [4] have also studied the interactions of fluids in a heat exchanger with the introduction of a supercritical fluid. The analysis was executed by varying mass flow rate and gauging the outlet temperatures. Jyothiprakash et al. [6] uses finite element methods to estimate the heat transfer in a multi fluid heat exchanger. A literature review on the above-mentioned papers and the inherent nature of supercritical fluids indicates the need for analysis of a two-fluid heat exchanger with a supercritical fluid as the cold fluid stream using finite element methods. In this study, an attempt is made to analyze the performance of a heat exchanger when the characteristics of varying c_p values with temperature are accommodated in a supercritical fluid stream.

2. GOVERNING EQUATIONS

For formulating the governing equations required, co-current and counter co-current flow arrangements are considered as per Fig1. And Fig.2. Several assumptions are made to simplify the study, which are as follows-

1. The heat exchanger operates under steady state conditions.
2. Heat loss to the surrounding is negligible.
3. There is no heat generation within the fluid streams.
4. The temperature distribution at the inlet of each fluid stream is uniform.
5. The overall heat transfer coefficients and the thermo-physical properties of all fluid streams are constant except the isobaric heat capacity of the supercritical fluid stream.

Applying the law of conservation of energy principle to both the fluids, the following equations are derived.

Hot Fluid:

$$\frac{dT_h}{dx} + \frac{UP}{C_h}[T_h - T_c] = 0 \tag{1}$$

Cold Fluid:

$$\frac{dT_c}{dx} \mp \frac{UP}{C_c}[T_h - T_c] = 0 \tag{2}$$

The boundary conditions are given by specifying the inlet temperatures of the two fluids, as indicated in Table 1.

Table 1: Boundary and operating conditions.

Hot fluid inlet (°C)	Cold fluid inlet(°C)	Hot fluid cp (kJ/kgK)	Overall heat transfer co-efficient U (W/m^2K)	Pressure of sCO_2 stream, considered as cold fluid (MPa)
70	23	4.187	500	7.5

The variation of c_p with the temperatures in a supercritical fluid is accommodated using the thermodynamic property table enlisted in National Institute of Standards and Technology, Chemistry Webbook [5]. Fig.3 illustrates the variation of c_p of supercritical carbon di-oxide with temperature, at a pressure of 7.5 MPa.

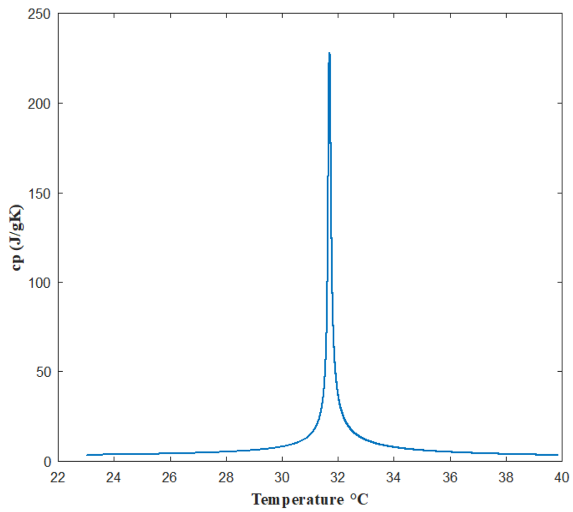

Fig.3 Variation of c_p with temperature

2.1 Finite Element Method

Despite the high possibility of arriving at analytical solutions for many heat transfer problems, there are several cases involving both boundary conditions and geometric parameters, where the possibility of determining the analytical solutions is low. One such case is the formulation of heat transfer in heat exchangers, where deducing the analytical solution is complex. Hence the numerical approach of Finite Element method is used in the present investigation to estimate the exit temperatures. This method is based on the discretization of the system into several small, interconnected sub-regions or elements, giving a piece-wise approximation to the governing equations. This is done to reduce the partial differential equations into a linear or non-linear algebraic expression. The governing equations (1) and (2) formulated above are converted to the algebraic form using weighted residual methods, which are as follows,

$$\int_0^1 W\left[\frac{dT_h}{dx} + Ntu \quad [T_h - T_c]\right] dx = 0 \tag{3}$$

$$\int_0^1 W\left[\frac{dT_c}{dx} \pm Ntu \quad R\left[T_h - T_c\right]\right] dx = 0 \tag{4}$$

Assuming linear variation of the hot and cold fluid temperatures in a single element, we get:

FOR CO-CURRENT FLOW

$$\left.\begin{aligned} T_h &= N_1 T_{hi} + N_2 T_{ho} \\ T_c &= N_1 T_{ci} + N_2 T_{co} \end{aligned}\right\} \tag{5}$$

FOR COUNTER-CURRENT FLOW

$$\left.\begin{aligned} T_h &= N_1 T_{hi} + N_2 T_{ho} \\ T_c &= N_2 T_{ci} + N_1 T_{co} \end{aligned}\right\} \tag{6}$$

Where

$$N_1 = 1 - \frac{x}{L} \; ; \; N_2 = \frac{x}{L}$$

Substituting these approximations in equations (3) and (4), a set of equations can be obtained if W is defined. In the methodology, the Galerkin's weighted residual method is used to estimate the value of W, in which the shape functions N_1 and N_2 are taken as weighted parameters. For a single element the discretized equations can be written in matrix form as:

$$[\mathrm{K}]\{T\} = \{f\} \tag{7}$$

Where [K] is the stiffness matrix, {T} is temperature matrix and {f} is the load vector.

$$\{T\} = \left[K_{global}\right]^{-1}\{f_{global}\} \tag{8}$$

The present investigation uses the marching technique, where the boundary conditions are applied right from the first element. Using the boundary conditions in the formulation, the exit temperatures are estimated for each element using equation (8). The exit temperatures of the first element become the inlet temperatures for the next element in their respective directions. This is iterated for all the elements along the length of the heat exchanger as shown in Fig.4, thus obtaining the exit temperatures along the heat exchanger using a program written in MATLAB®.

Table 4 and Table 5 present the convergence study of the current investigation, where the values of cold fluid exit temperatures and the maximum c_p converge up to four decimal places. This convergence is observed when the length of the heat exchanger is discretized into 10500 and 10600 elements, for co-current and counter co-current flow arrangements respectively. These converging exit temperatures and c_p values are used to estimate the performance parameters of the heat exchanger in the analysis.

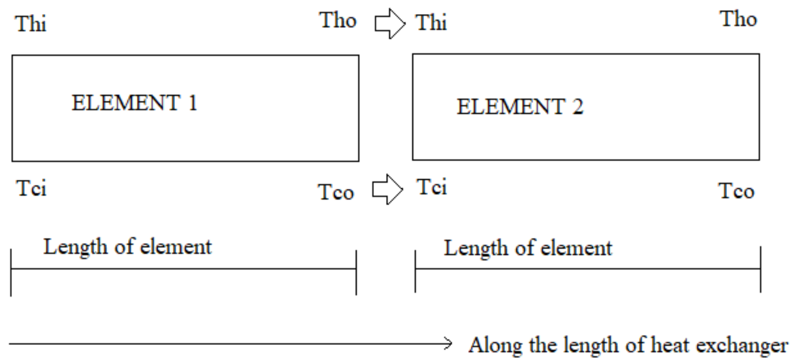

Fig.4 Schematic of marching technique

3. RESULTS AND DISCUSSIONS

In the current investigation, the cold fluid is taken as supercritical carbon di-oxide (sCO_2) and the hot fluid stream is water, with the inlet conditions as specified in Table.1 for both co-current and counter co-current flow arrangements. The validation of the methodology is illustrated in Fig.5 and Fig.6 for these arrangements. For both the cases, we observe a region in the sCO_2 temperature plot where the values of temperature vary minimally with the length of the heat exchanger. This corresponds to the supercritical regime of carbon-dioxide, attained after reaching a temperature of 30.98°C. Fig.3 demonstrates the spike in c_p values for carbon-dioxide in the supercritical region. This spike in c_p values is commensurate with a small range of temperatures (30°C - 34°C). This trend is accommodated in the present investigation and is validated in the Fig.5 and Fig.6 for co-current and counter co-current flow arrangements respectively.

The points of interest in Fig.5 are the temperatures of sCO_2 between the length 0.15m to 0.5m along the heat exchanger. In this part of the heat exchanger, the temperature of cold fluid stream remains varies minimally, and the values of c_p for sCO_2 increase drastically for these temperatures. This is because the sCO_2 fluid stream enters the supercritical region, where the c_p values are unusually high. We observe a similar trend for the counter co-current flow arrangement in Fig.6, where the temperature varies minimally between the length of 0.6m to 0.85m, due to the opposite flow direction of cold fluid stream. A maximum c_p of 228.0463 J/gK is observed at a temperature of 31.71°C, in the co-current flow arrangement. A maximum c_p of 228.2663 J/gK is observed at a temperature of 31.73°C, in the counter co-current flow arrangement. To judge the performance of the heat exchanger in both configurations, the effectiveness-NTU method is adopted in the study. The exit temperature plots (Fig.7 and Fig.8) are used to gauge the effectiveness of the heat exchanger. The thermal effectiveness of a heat exchanger ε is defined as the ratio of the actual rate of heat transfer to the maximum possible heat transfer in a heat exchanger.

$$\varepsilon = \frac{\dot{Q}_{\text{act}}}{\dot{Q}_{\text{max}}}$$

In terms of the amount of heat lost by the hot fluid we get $Q_{act} = C_h (T_{hi} - T_{ho})$

In terms of the amount of heat gained by the cold fluid we get $Q_{act} = C_c (T_{co} - T_{ci})$

To compare the heat capacity of the hot fluid (C_h) with the heat capacity of the cold fluid (C_c), a single value ($\overline{cp}$) for the heat capacity of sCO_2 is required. This value is deduced using the integral mean c_p method, as per equation (9).

$$\overline{cp} = \frac{1}{Texit-Tin}\int_{Tin}^{Texit} cp.\,dT \qquad (9)$$

The integrated mean c_p value for the co-current flow and counter co-current flow is found to be 9.5335 J/gK, and 9.7391 J/gK respectively. This integrated mean c_p is multiplied with the mass flow rate of cold fluid stream to obtain C_c. For the present analysis, it is to be noted that C_h remains lesser than C_c for both fluid flow arrangements. Hence,

$$C_{min} = C_h \qquad (10)$$

$$Q_{max} = C_h\,(T_{hi} - T_{ci})$$

Using the exit temperature plots, the effectiveness of the heat exchanger for co-current and counter co-current flows is estimated using equations (11) and (12) respectively.

$$\varepsilon = \frac{T_{hi}-T_{ho}}{T_{hi}-T_{ci}} \qquad (11)$$

$$\varepsilon = \frac{T_{hi}-T_{ho}}{T_{hi}-T_{co}} \qquad (12)$$

The NTU is estimated using equation (13)

$$NTU = \frac{U\,A}{Cmin} \qquad (13)$$

Fig.7 and Fig.8 illustrate the variation of the hot fluid and cold fluid temperatures along the length of the heat exchanger for both the arrangements considered. The hot fluid (water) enters the heat exchanger at a temperature of 70 °C, and the supercritical $sC0_2$ stream enters at an inlet temperature of 23 °C. The trend of the hot and cold fluid temperatures resembles a generic co-current and counter co-current flow temperature distribution in Fig.7 and Fig.8 respectively. The final exit temperatures of the fluid streams have been presented in Table 2. The increase of temperature in the cold stream and reduction of temperatures in the hot stream denotes the heat loss from the hot water stream and heat gain by the sCO_2 stream.

Table 2 Performance parameters of co-current and counter co-current arrangements

Sl. No	Property	Co-current flow		Counter co-current flow	
		Unbalanced mass flow rate (0.8837 kg/s for hot fluid stream and 0.6 kg/s for cold fluid stream)	Balanced mass flow rate of 0.6 kg/s for both streams	Unbalanced mass flow rate (0.8837 kg/s for hot fluid stream and 0.6 kg/s for cold fluid stream)	Balanced mass flow rate of 0.6 kg/s for both streams
1	NTU	0.1350	0.1990	0.1350	0.1990
2	Effectiveness	0.6854	0.5545	0.8659	0.9036

The performance parameters gauged from the exit temperature plots (Fig.7 and Fig.8) are presented in Table 2. The value of effectiveness in the counter co-current arrangement is observed to be higher than the effectiveness value in a co-current arrangement, owing to a more uniform temperature difference between the fluid streams in the former case. NTU remains the same for similar mass flow rate conditions in both the arrangements, as C_{min} is the same in all cases as per equation (10).

To demonstrate the performance enhancement of the heat exchanger when a supercritical fluid is introduced, the study compares the performance of a heat exchanger with a regular fluid (water) to the performance of a heat exchanger with sCO_2 fluid stream. Fig. 9 and Fig. 10 are the exit temperature plots of the heat exchanger when water is introduced as the cold fluid stream, instead of supercritical carbon-dioxide for the two considered arrangements. The working conditions of the heat exchanger are taken to be the same, when the above-mentioned comparison is made. Using equations (11) and (12). The effectiveness is estimated using equations (11) and (12), to analyze the performance of the heat exchanger with different cold fluid streams. The values are tabulated in Table 3. There is a substantial improvement in the effectiveness of the heat exchanger when the cold stream is taken to be sCO_2, in both co-current and counter co-current arrangements. The NTU values are constant for all cases as the value of C_{min} remains the same for all working conditions. The increased effectiveness of a heat exchanger with sCO_2 stream is clear indication of the fact that the performance of the heat exchanger improves when a supercritical fluid stream is introduced.

Table 3 Performance parameters for the comparison between water and sCO_2 as cold fluid stream (balanced mass flow rate condition)

Sl. No	Property	Co-current flow		Counter co-current flow	
		Water as cold fluid stream (c_p = 4187 J/kgK)	sCO2 as cold fluid stream ($\bar{c}_p$=9540 J/kgK)	Water as cold fluid stream (c_p = 4187 J/kgK)	sCO2 as cold fluid stream ($\bar{c}_p$=9540 J/kgK)
1	NTU	0.1990	0.1990	0.1990	0.1990
2	Effectiveness	0.5093	0.5545	0.6665	0.9036

Table 4 Convergence study using Finite element method (co-current flow arrangement)

Sl. No	Property	Number of elements						
		200	500	1000	2000	4000	10500	10600
1	Tc exit (℃)	40.0307	39.9243	39.8903	39.8736	39.8652	39.8601	39.8601
2	cp max	227.7132	227.8108	227.944	228.0111	228.0447	228.0463	228.0463

Table 5 Convergence study using Finite element method (counter co-current flow arrangement)

Sl. No	Property	Number of elements						
		200	500	1000	2000	4000	10500	10600
1	Tc exit (°C)	41.0176	39.9492	39.9105	39.8931	39.8714	39.8802	39.8802
2	cp max	227.9388	228.0309	228.164	228.2350	228.2637	228.2663	228.2663

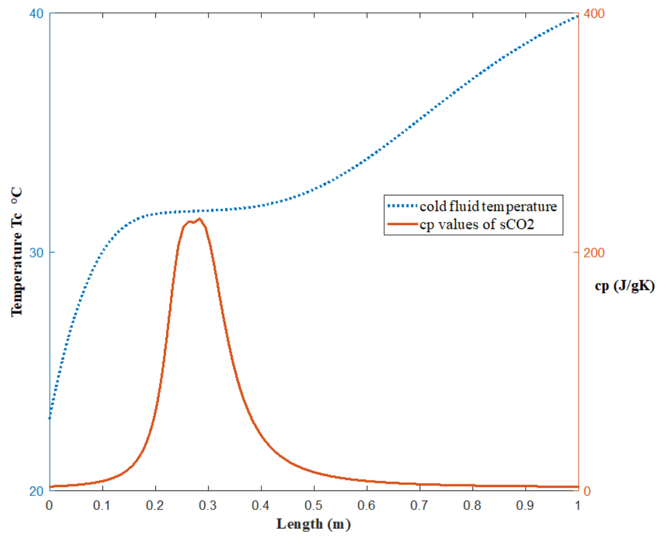

Fig.5 Temperature of sCO2 and corresponding cp values vs length of the heat exchanger (co-current)

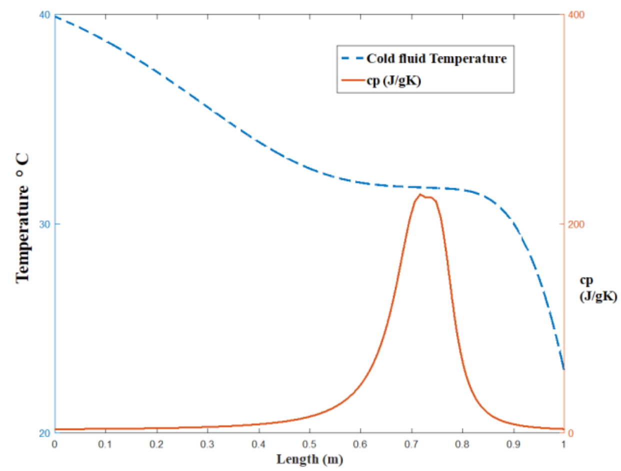

Fig.6 Temperature of sCO2 and corresponding cp values vs length of the heat exchanger (counter co-current)

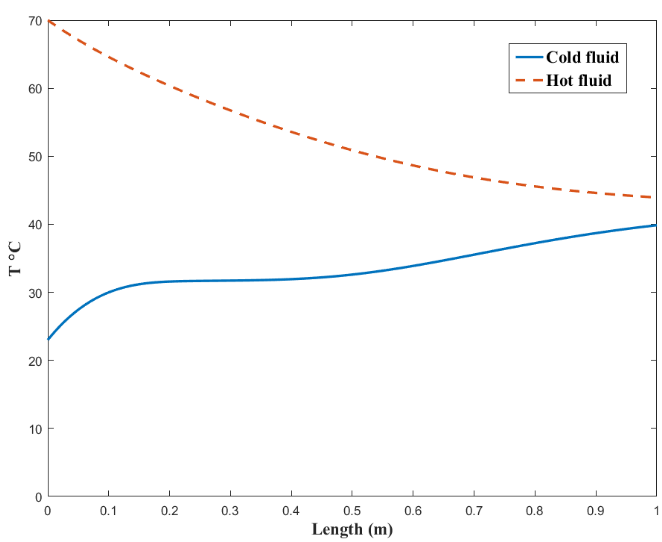

Fig.7 Fluid temperatures vs length (co-current arrangement)

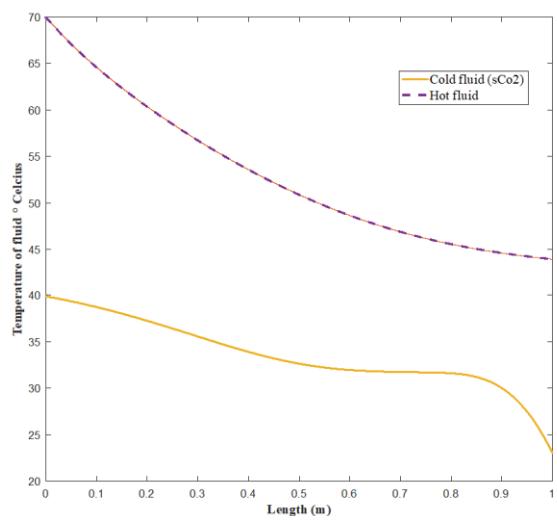

Fig.8 Fluid temperatures vs length (counter co-current arrangement)

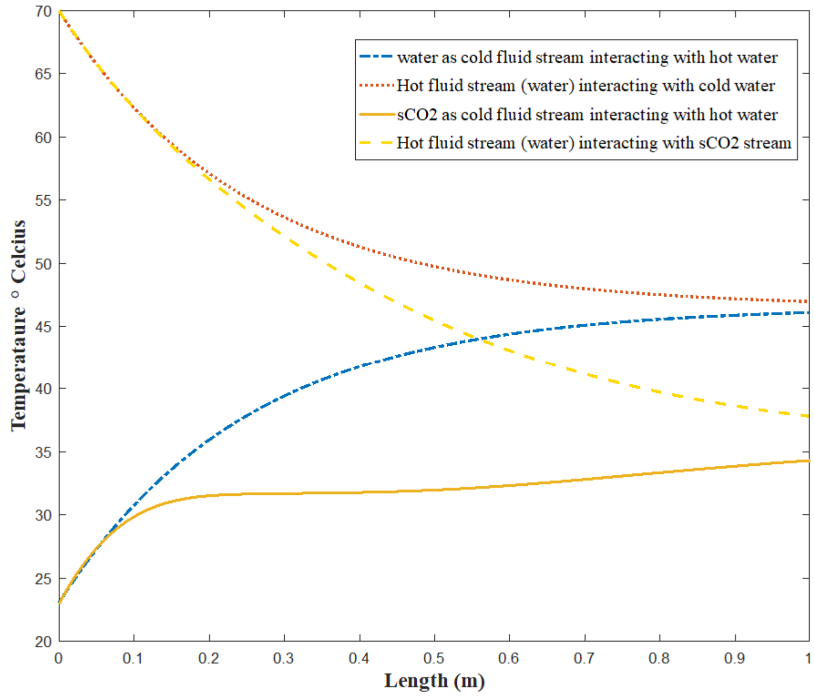

Fig.9 Temperature plots of heat exchanger with both fluids as water, and heat exchanger with sco2 and hot water vs length of the heat exchanger (co-current flow).

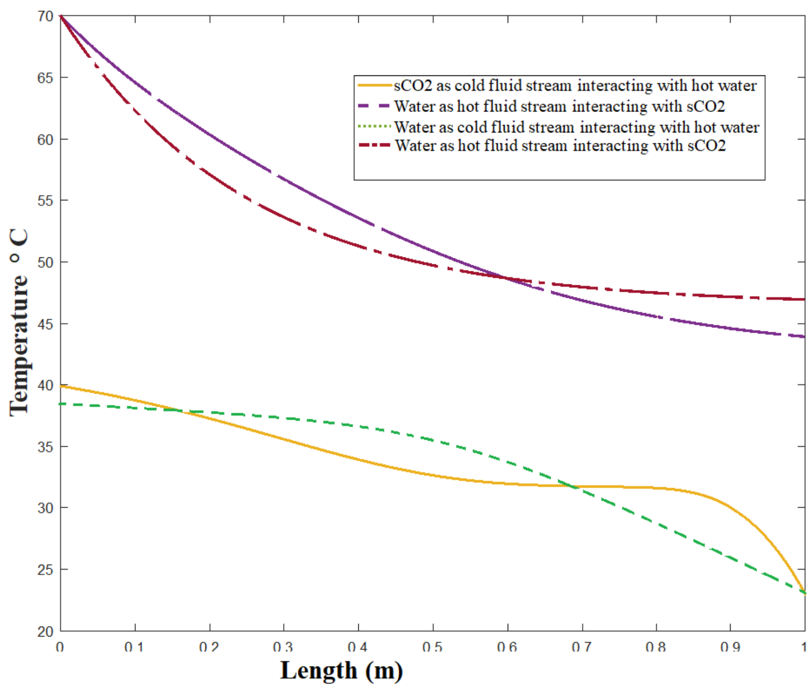

Fig.10 Temperature plots of heat exchanger with both fluids as water, and heat exchanger with sco2 and hot water vs length of the heat exchanger (counter co-current flow).

4. CONCLUSIONS

The performance of a two-fluid heat exchanger for both co-current and counter co-current flow configurations was analyzed when a supercritical fluid stream of carbon di-oxide was introduced as the cold stream. Finite element method was used to formulate the governing equations required to emulate the heat transfer. A sudden spike in the c_p values for the supercritical stream was observed in both the configurations, when the temperatures of the cold fluid stream corresponded to the supercritical region of CO_2. This spike of c_p values was accommodated in the mean integral c_p value, which was used to determine the NTU of the heat exchanger. Effectiveness was gauged using the exit temperature plots generated (Fig.7 and Fig.8). The effectiveness of the heat exchanger for a counter co-current flow was substantially higher than the co-current flow (as per Table 2). On comparing the performances of a heat exchanger with sCO2 and water as cold fluid streams, a remarkable increase in effectiveness is noted (Table 3) in the case of sCO_2 considered as cold fluid stream, supporting the claim of improved performance on the introduction of supercritical fluids in a heat exchanger.

5. NOMENCLATURE

C_h	Heat capacity of hot fluid	NTU	Number of transfer units
C_c	Heat capacity of cold fluid	ε	Effectiveness of heat exchanger
U	Overall heat transfer co-efficient	$\overline{c_p}$	Integrated mean specific heat capacity
x	Direction of flow along the heat exchanger length	L	Length of the heat exchanger
W	Weighted residual	P	Contact perimeter

6. REFERENCES

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